The Role of Multipath in Antenna Height Tests at Table Mountain

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Abstract

In order to determine what effect the antenna height has on baseline horizontal and height components, data were collected on short (~5 meters) baselines at the Table Mountain Antenna Range. These data were analyzed, and height biases of 1.0 to 1.7 cm were found between baselines with high and low antenna setups when tropospheric parameters were estimated. Horizontal components did not change. When both ends of the baseline were occupied with the same height setups, no bias was seen, although the repeatabilities for low-low antenna setups were worse than for high-high setups. This paper will show that the height bias in high-low antenna baselines is the result of multipath at the low antenna being mismodeled as troposphere. First we construct a simplified model of multipath phase and amplitude error as a function of antenna height. To verify this model we compare the predicted amplitude pattern for low and high antenna setups to the signal-to-noise ratio recorded by the GPS receivers. We then show that the multipath phase error generated by a low antenna setup correlates to a tropospheric signal for large intervals of elevation. In specific, the L1 phase error for an antenna height of 24 cm shows a strong correlation to a tropospheric delay in the elevation range of 17.5 to 29.0 degrees. This range of elevation is sensitive to both multipath and tropospheric errors. We also see that multipath from near ground reflections have a long period (> 1 hour) which makes them less likely to average out over time. We then briefly discuss the possibility of modeling and/or identifying the multipath in the low antenna setup. Based on our simple multipath model and experimental results, we conclude for environments such as Table Mountain that: (1) GPS antennas should not be placed near the ground because the multipath phase error is of low frequency and can be mismodeled as a tropospheric delay, (2) The multipath from the low antenna setup can not be easily modeled or corrected, and (3) GPS antennas mounted on tripods 1.5 meters above the ground suffer from high frequency multipath that can not be modeled as a tropospheric parameter and therefore does not effect the baseline determination.

Introduction

UNAVCO held working group meetings in May of 1995 to discuss issues and concerns in the area of using the Global Positioning System (GPS) for Earth sciences. The focus of working group 3 (WG3) was on the Technology and Development section of UNAVCO and identified issues and problems that should be addressed by this group. One of the resulting action items of the WG3 meeting was to test and evaluate various methods for mounting GPS antennas over geodetic monuments. Within the last year, alternatives to the tripod/optical plumb combination have been proposed and used, such as the JPL spike mount, and there were concerns that different antenna configurations may effect the results of baseline measurements.

In order to investigate the effect of low antenna setups (< 0.5 meters from the ground) on baseline horizontal and height components, UNAVCO set up a test site at Table Mountain where antennas could be easily mounted near to the ground. Monuments were then occupied with high and low antenna setups with various GPS receivers and antennas. The results from occupations using Trimble SSE GPS receivers and Trimble 4000SST L1/L2 GEO (SST) antennas showed height biases of up to 1.7 cm in baselines between high and low antenna setups when tropospheric parameters were estimated.

Antenna Height Tests at Table Mountain

Four corners of a 5 m square were occupied alternately by high and low setups (see Figure 1) using Trimble SSE receivers and Trimble SST antennas. The high antenna setups consisted of antennas mounted on survey tripods giving an antenna height in the range of 1.5 meters. The low setups consisted of constant height tribrachs mounted directly to the bench marks, providing an antenna height of about 9 cm above the monuments reference point (see Figure 3). The benchmarks of the 5 meter square consist of stainless steel rods set in concrete (see Figure 2). The reference point for the steel rod monuments is a dimple placed in the center of the top of the rod. Each of the monument rods rise about 10 cm above the ground and thus the height of the antennas phase center above the ground is about 25 cm for setups using the constant height tribrachs (the phase center of SST antenna is ~7 cm above the base of the antenna).

Figure 1. Table Mountain test site.

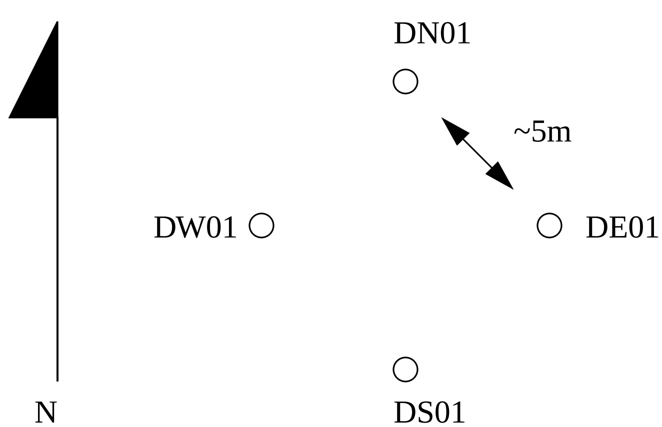


Figure 2. Monuments at Table Mountain test site.

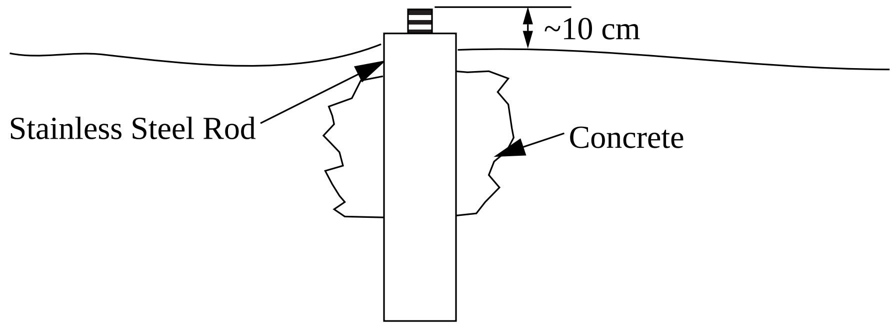
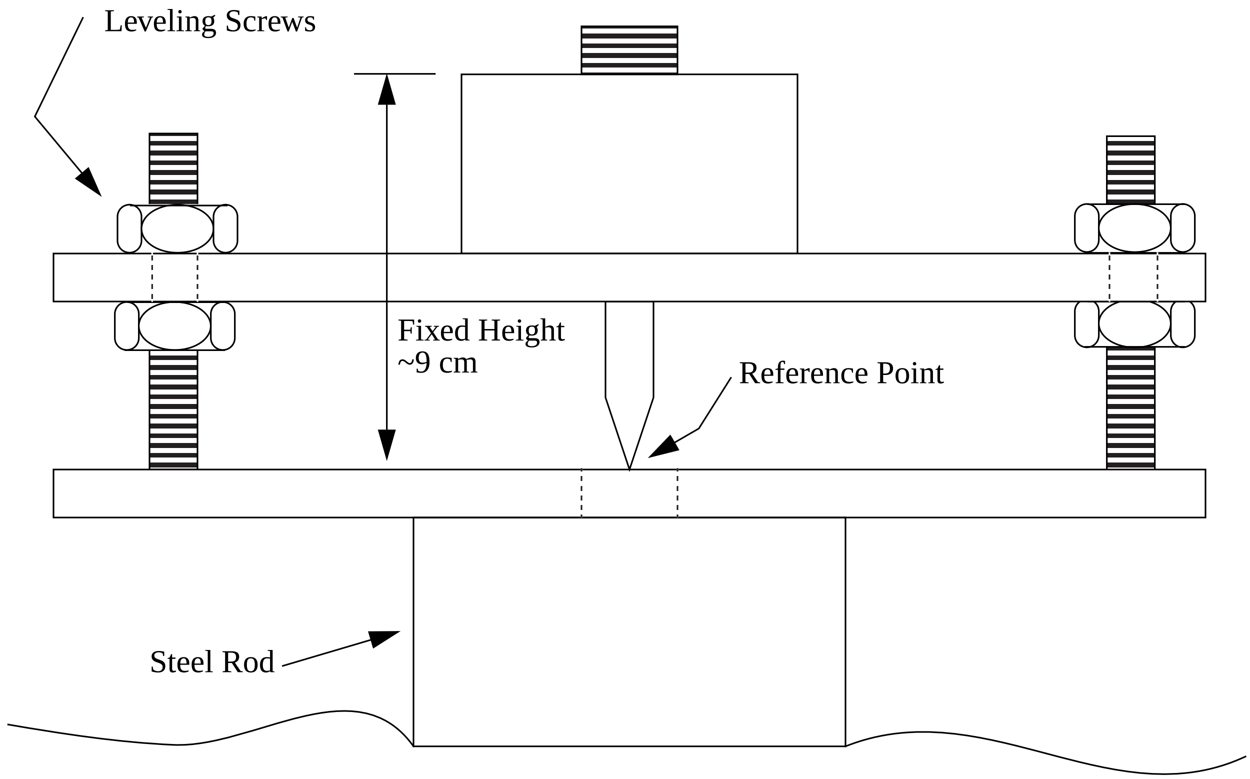


Figure 3. Sketch of the fixed height tribrach. The fixed height tribrach has a spike welded to the bottom center of the top plate, the bottom of which rests in the reference dimple of the steel rod monument. There are three leveling screws (only two are shown in the above sketch) which can be adjusted to make the antenna level.



The SSE receivers were setup to collect data at a rate of 30 seconds for 23 hours a day, beginning at 00:00 UTC. The elevation mask used for data processing was 15 degrees and the Bernese Version 3.6 GPS processing package was used for data analysis. Two types of solutions were generated for the L1, L2 and L3 frequencies, a non-tropospheric solution where tropospheric parameters were not estimated and a tropospheric solution where a zenith tropospheric delay was estimated on an hourly basis (see Table 1). For very short baselines such as these, there is virtually no difference in the path that GPS signals take through the troposphere to each of the antennas and no estimation of troposphere is required to obtain baseline results. In fact, estimating the additional troposphere parameter actually weakens the solution for very short baselines. We computed solutions with troposphere corrections because in most geodetic applications, baseline lengths are long enough that tropospheric estimation is required, and we wish to verify high and low antenna setups for the type of processing that is most commonly used.

On day 159 of 1995, the monument DE01 was occupied with a fixed height tribrach and DW01 was occupied with a tripod setup. The height of the phase center above the ground for DE01 was about 24 cm while for DW01, it was about 150 cm. The difference between the L3 solution with and without tropospheric estimation was greater than 1 cm. The L1 solution without tropospheric estimation was within 3 mm of the ground truth measurement (done with conventional survey equipment, accurate to 2 mm). The L1 solution agreed with previous and subsequent L1 solutions without tropospheric estimation, regardless of the antenna heights at DE01 and DW01. The fact that solutions performed without tropospheric estimation agree rules out blunders such as mismeasured antenna heights. For occupations where DE01 is occupied with a high antenna setup and DW01 is occupied with a low antenna setup, an equal but opposite height bias is observed.

Table 1: High-Low Height Solutions, DE01-DW01 day 159

Freq	No Tropospheric Est.	Tropospheric Est	Ground Truth
L1	0.1471 m (-2.9 mm)	0.1521 m (+2.1 mm)	0.150 m
L2	0.1481 m (-1.9 mm)	0.1489 m (-1.1 mm)	
L3	0.1457 m (-4.3 mm)	0.1571 m (+7.1 mm)	

Table 1 that shows various solutions for the DE01-DW01 baseline on day 159. Each column of the table shows results for each frequency type with and without tropospheric estimations. The first column (No Tropospheric Est.) are the solutions without hourly tropospheric estimations and the second column (Tropospheric Est.) is with hourly tropospheric estimates. The third column shows the ground truth as measured by standard surveying techniques. The values in parenthesis shows the difference from the ground truth value.

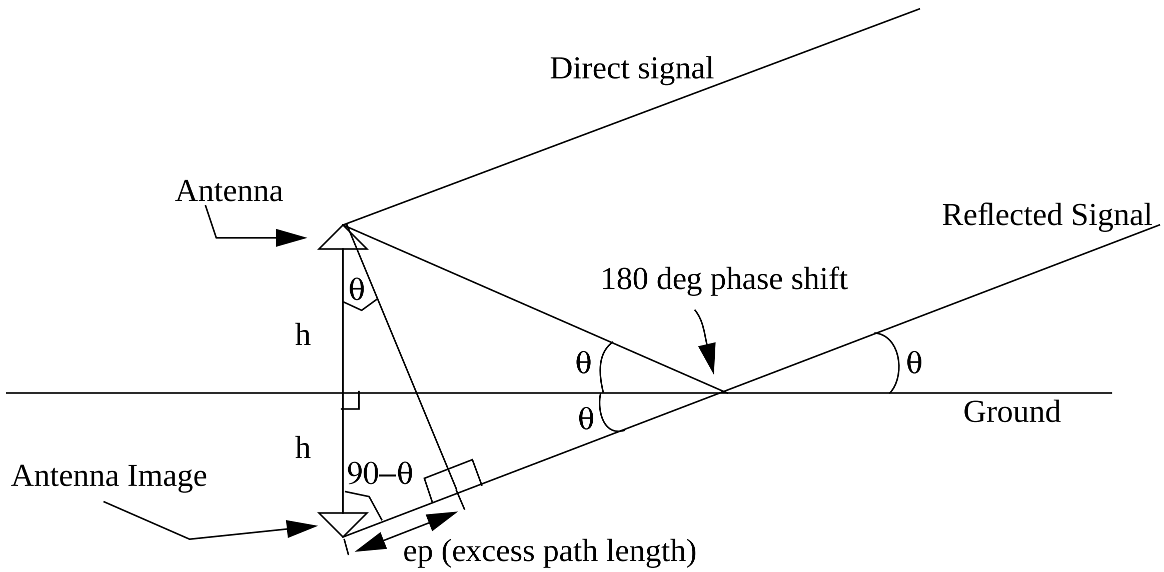
The difference in the L3 solutions is 1.14 cm. The average of the one hour zenith correction values is 3.53 mm which is about 1/3 the height change seen. Since an error in the zenith tropospheric delay causes a height error three times higher [e.g. *Herring*, 1986; *Beutler et al.*, 1987], this indicates that the difference between the two solutions is due primarily to mismodelling of the troposphere.

Since the actual tropospheric delay between the two stations is zero, there must be some error source that does not cancel between the high and low antenna setups. Since the characteristics of multipath will be different for high and low setups, we now turn our attention to how multipath contaminates GPS measurements.

Simple Multipath Model:

Multipath is the effect of mixing direct and reflected signals from the GPS satellites [e.g., *Elosegui et al.*, 1995; *Georgiadou et al.*, 1988; *Clark*, 1992]. The Table Mountain test site is located on a high mesa. There is only one building near the test site, about 150 meters to the S-SE. We will consider a simple model where the only reflected signals come from the ground (see figure Figure 4).

Figure 4. Diagram of multipath geometry.



From the above figure, we can derive the following relations:

$$ep = 2h\sin\theta \tag{1}$$

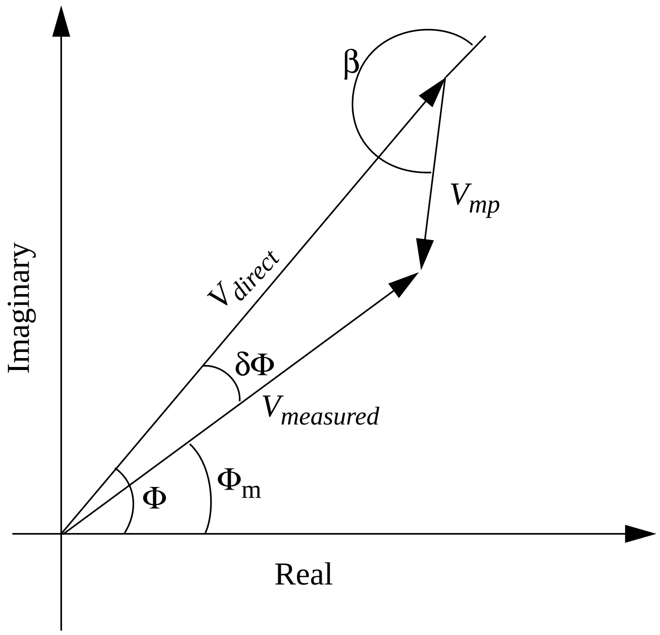
$$\beta = \frac{ep}{\lambda} \times 2\pi + \pi = \frac{4\pi h \sin\theta}{\lambda} + \pi \tag{2}$$

Where:

ep = excess path length of reflected signal.
h = height of antenna above the ground.
θ = elevation angle of satellite.
β = phase change of reflected signal.
λ = wave length of signal.

To visualize the effect of adding the reflected multipath to the direct signal, it is helpful to use a phasor diagram, as in Figure 5.

Figure 5. Phasor diagram of mixing direct and multipath signals.



$$V_{direct} = A_0 e^{j\Phi} \tag{3}$$

$$V_{mp} = \alpha A_0 e^{j(\Phi + \beta)} \tag{4}$$

$$V_{measured} = V_{direct} + V_{mp} \tag{5}$$

Notation for Figure 5:

Φ = phase of incoming GPS signal.

Φ_m = phase value actually measured.

$\delta\Phi$ = phase error caused by multipath.

V_{direct} = direct signal phasor.

V_{mp} = multipath signal phasor.

$V_{measured}$ = measured signal phasor.

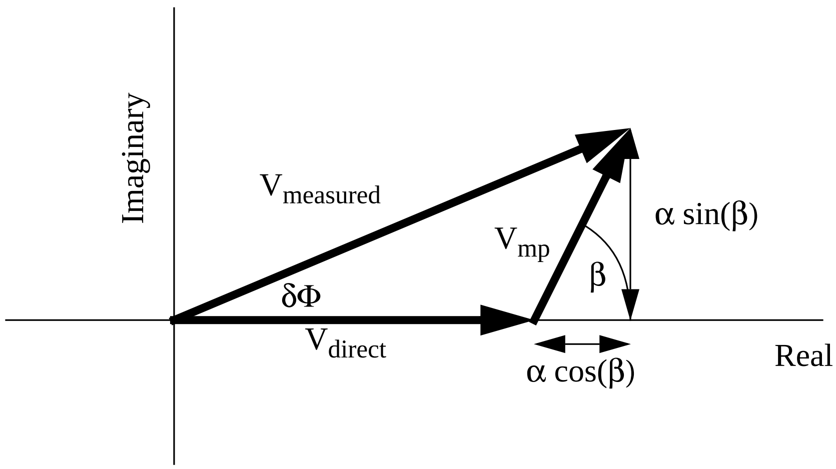
A_0 = amplitude of direct signal.

α = attenuation factor of reflected signal.

As β changes, the multipath vector ‘spins’ around the tip of the incoming direct signal, inducing both positive and negative phase errors. Note that the actual phase error is dependent on both the phase of the multipath signal and its amplitude.

Since we are only interested in the phase error $\delta\Phi$, we can rotate the vectors in Figure 5 so that V_{direct} is along the Real axis as in Figure 6. If we set A_0 to 1, we then can calculate the amplitude of the measured signal relative to the amplitude of the direct signal. The amplitude and phase error of our measured signal $V_{measured}$ are then:

Figure 6. Calculation of amplitude and phase error of the measured GPS signal. Here we assume that $A_0 = 1$.



$$Re(V_{measured}) = 1 + \alpha\cos\beta \tag{6}$$

$$Im(V_{measured}) = \alpha\sin\beta \tag{7}$$

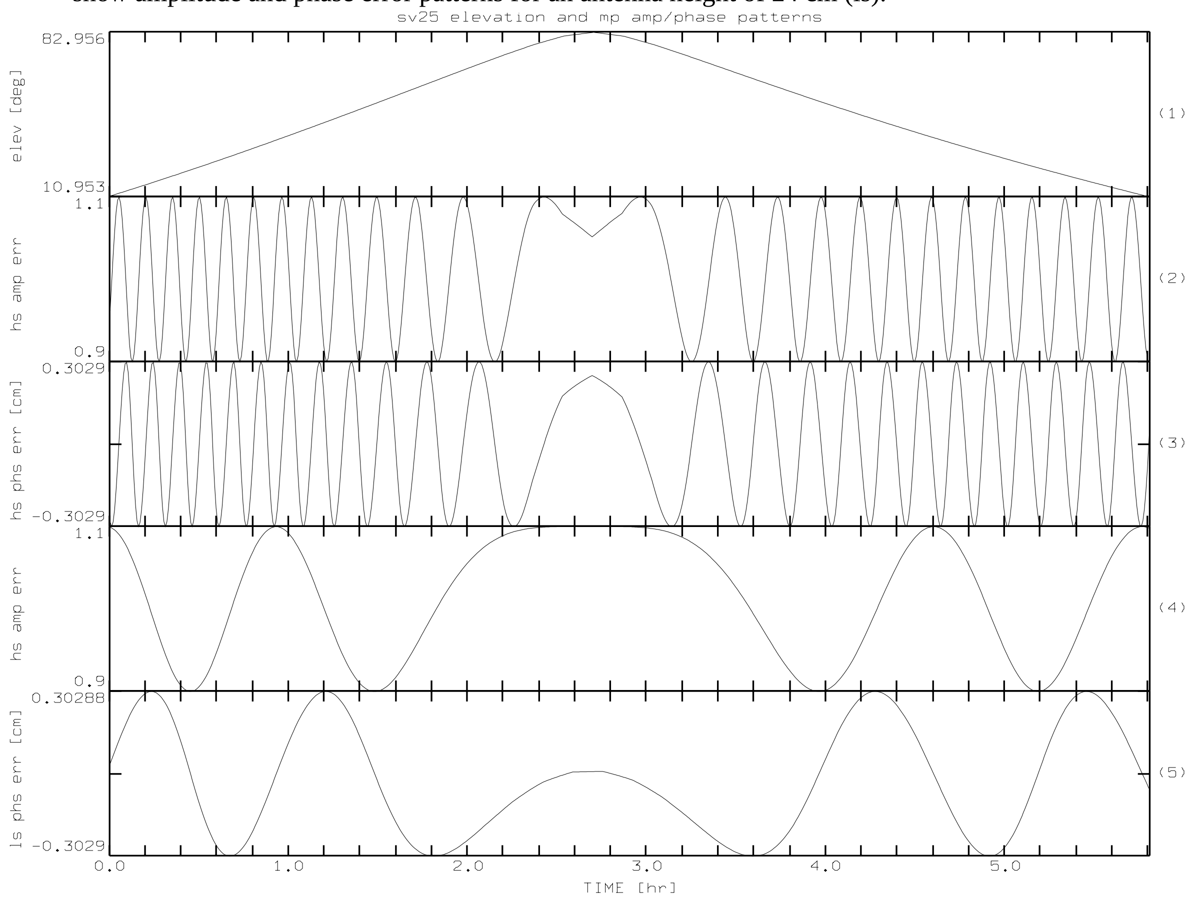
$$|V_{measured}| = \sqrt{Re(V_{measured})^2 + Im(V_{measured})^2} \tag{8}$$

$$\delta\Phi = \mathrm{atan}\left(\frac{Im(V_{measured})}{Re(V_{measured})}\right) \tag{9}$$

First Order Multipath Amplitude and Phase Patterns

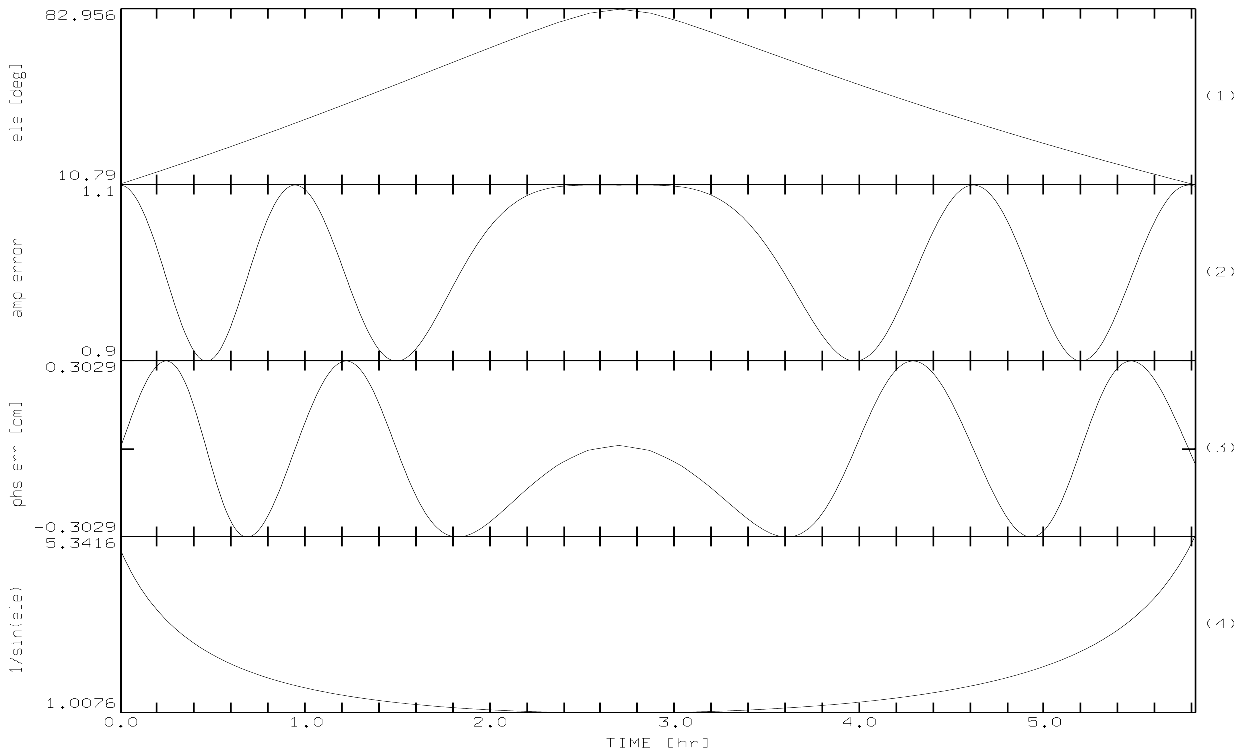
For a very simple model of how ground reflections affect the amplitude and phase of measured GPS signals, we will assume that reflections come from a flat ground and the reflected signal is attenuated (α) by a fixed amount [e.g. *Elosegui*, 1995]. The graph below (Figure 7) was generated from an actual elevation track of a satellite over the Table Mountain test area. For this example, we used a value of 0.1 for α and 1.0 for A_0. The top trace shows the elevation of the satellite, trace 2 the amplitude of the measured signal for an antenna height of 1.5 m, trace 3 the phase error for an antenna height of 1.5 m, trace 4 the amplitude of the measured signal for an antenna height of 24 cm and trace 5 the phase error for an antenna height of 24 cm.

Figure 7. Amplitude and phase patterns for a satellite following the elevation track shown in trace 1. Traces 2 and 3 show amplitude and phase error patterns for an antenna height of 1.5 meters (hs) and traces 4 and 5 show amplitude and phase error patterns for an antenna height of 24 cm (ls).



The most striking feature of Figure 7 is that both the amplitude and phase error patterns oscillate much faster for the higher antenna set up. The maximum phase error is, however, the same. Figure 8 shows the amplitude and phase error patterns for a 24 cm antenna height along with a tropospheric signal with zenith amplitude of 1.0 cm.

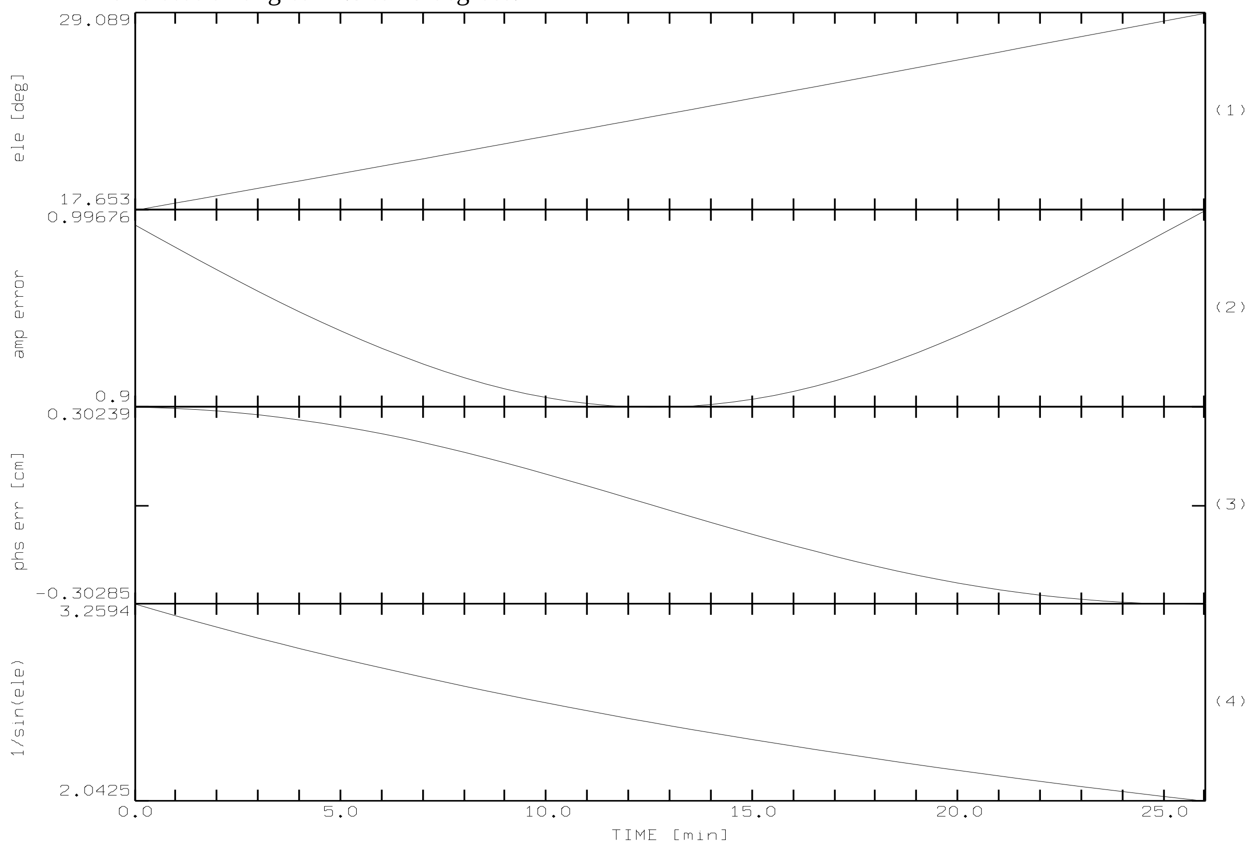
Figure 8. Amplitude and phase patterns for an antenna height of 24 cm for the elevation track show in trace 1. Trace 2 shows the measured amplitude, trace 3 shows the phase error and trace 4 shows a tropospheric delay with zenith intensity of 1.0 cm.



If we look at the phase error and tropospheric delay curve more closely (Figure 9) between the elevations of 17.5 and 29 degrees, we see that the phase error of the multipath is in the same direction and could be partially modeled as a tropospheric signal. For the case where the antenna height is 1.5 meters, the phase error changes so quickly with elevation that it can not be modeled by a realistic zenith tropospheric correction.

Figure 9 shows the traces in Figure 8 from the time period 00:18 to 00:42, where the elevation of the satellite is changing from 17.5 degrees to 29.0 degrees. The change in the multipath phase error is 0.6 cm, while the change in the tropospheric delay (assuming a 1 cm zenith delay) is 1.22 cm. A zenith delay of 0.82 cm would be required to give the same change as the change in the multipath phase error, which is much higher than the zenith correction that was estimated (the average estimated zenith correction was 0.353 cm). This is because the relative amplitude of the actual multipath signal is probably much weaker than that for the assumed model since the reflected signal is coming into the GPS antenna from the back plane of the antenna, where the gain is considerably lower than the gain at positive elevations. Also, all of the observations used for the tropospheric estimations do not come from satellites within the critical elevation range, and the tropospheric correction is the best fit to all the data observed.

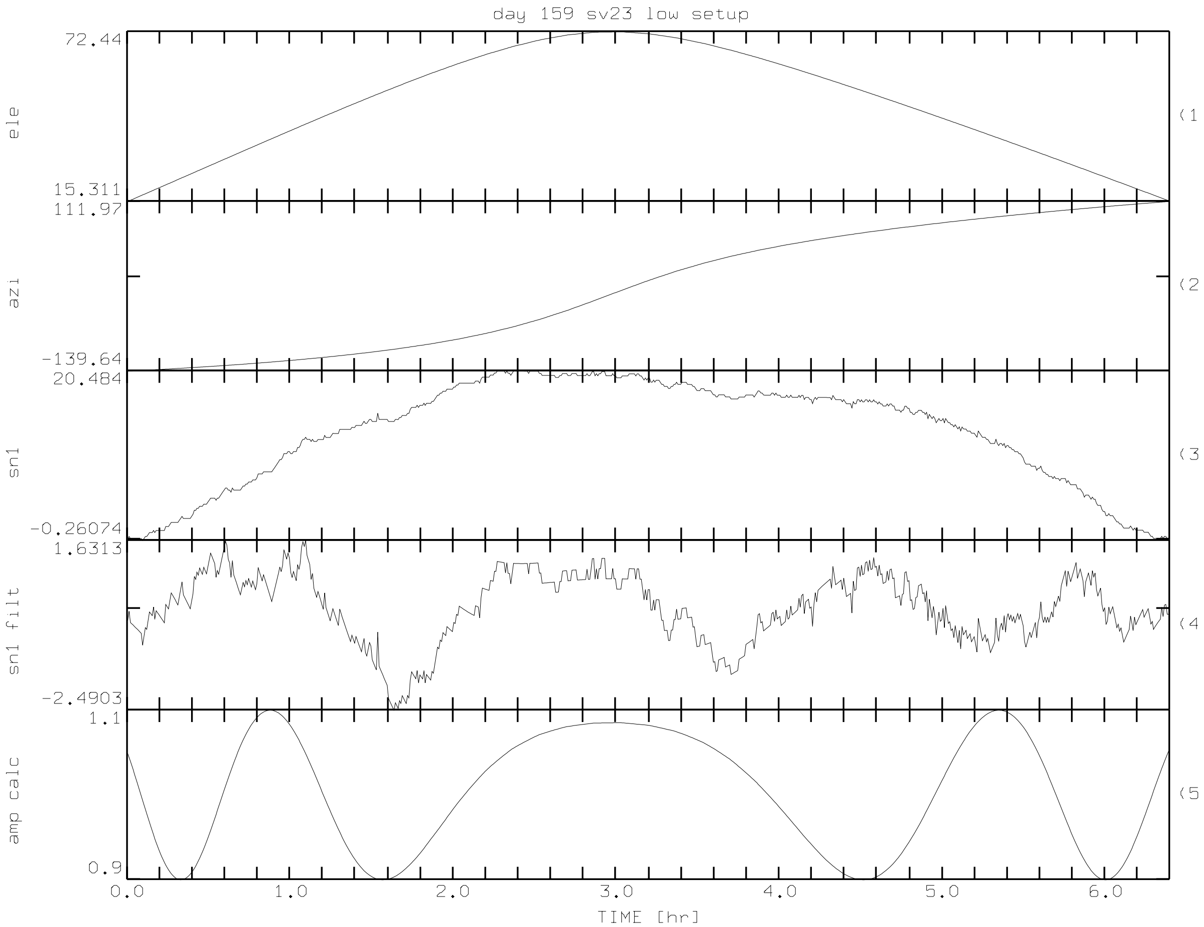
Figure 9. Amplitude and phase error patterns along with tropospheric delay for an antenna height of 24 cm for elevation angles 17.5 to 29 degrees.



If we look at the signal-to-noise ratios (SNR) recorded by the receiver, we can get an idea of what the actual amplitude pattern looks like. Figure 10 shows elevation, azimuth, L1 SNR, filtered SNR, and the synthetic amplitude for a 24 cm antenna height. The raw SNR observations, shown in trace 3, are dominated by the gain pattern of the GPS antenna. This pattern must be removed in order to see the change in SNR due to the multipath. In order to do this, the raw SNR trace was baseline corrected to force the SNR values to be zero when the satellite was rising through 15 degrees and setting through 15 degrees. Next the SNR values were run through a zero phase high-pass filter with a cutoff frequency of 0.0001 Hz (2.76 hrs). The baseline correction step is very important because the response of the highpass filter will be added to the result trace if the end-points of the time series are not zero. Since the SNR values were not exactly the same when the satellite rose through 15 degrees and set through 15 degrees, it was necessary to remove a line from the raw SNR data.

The filtered SNR values will still contain low frequency components of the GPS antenna gain pattern. A better approach would be to use the actual gain pattern of the antenna to remove the SNR signature of the antenna [e.q. *Axelrad et al.*, 1994; *Schupler et al.*, 1994]. This is currently not implemented at UNAVCO and would be time consuming and we only want to get a general understanding of what the amplitude error signal looks like. Care would also need to be taken in order to scale the measured amplitude pattern to measured SNR values. Also ignored are possible GPS receiver gain drifts.

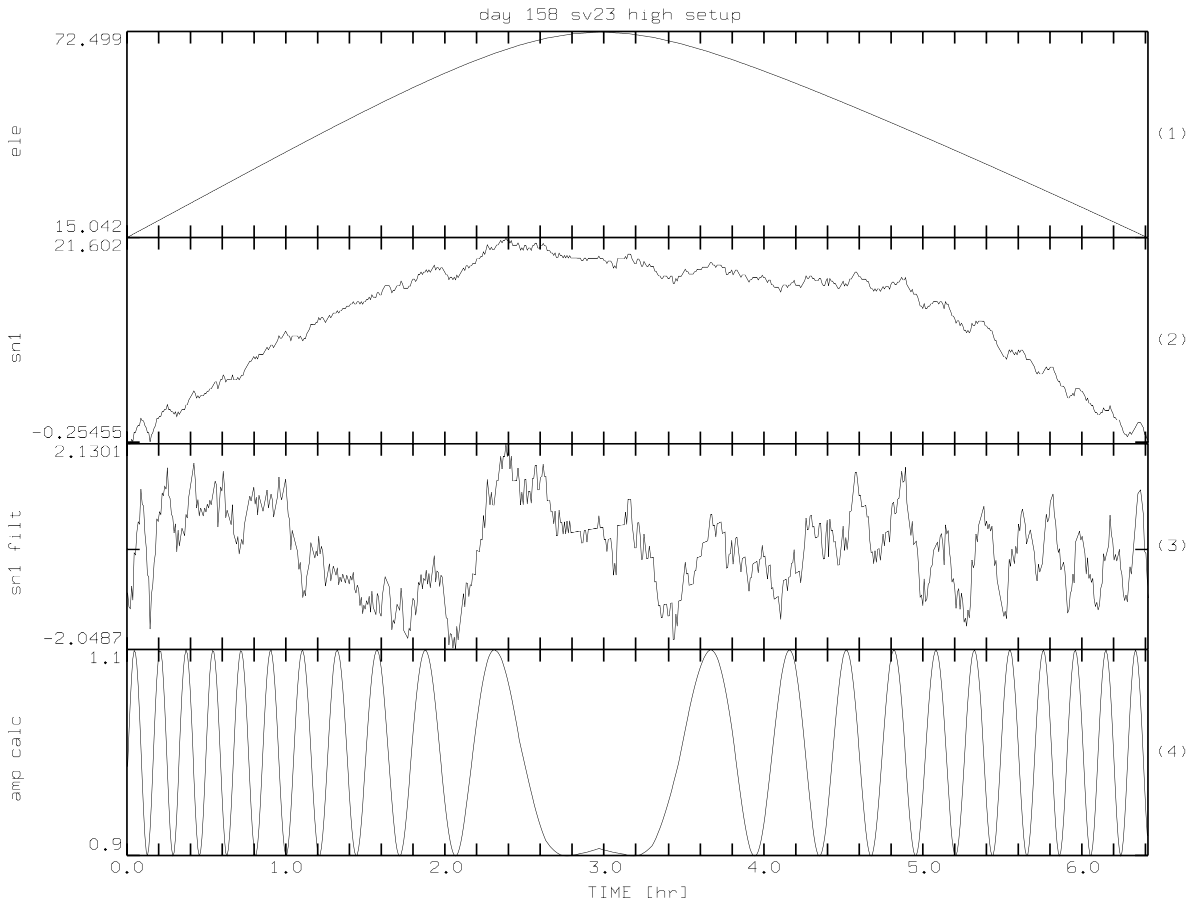
Figure 10. Observed SNR values for L1 as compared to our simple multipath model. Trace 1 shows the elevation of the satellite, trace 2 the azimuth, trace 3 the baseline corrected observed SNR, trace 4 the result after a highpass filter of the SNR and trace 5 the modeled amplitude.



The two traces we want to compare in Figure 10 are the filtered SNR values (trace 4) and the synthetic amplitude values (trace 5). The left hand side of the trace 4 matches fairly well up to the point where the satellite reaches its maximum. Notice that in trace 4 there are two minima points to the left of the maximum elevation point and 3 to the right. This indicates that the effective antenna height is increasing as the satellite crosses the sky. The ground around the marker is actually not completely flat and in the region within one meter of the marker it can vary by 10 cm or more in the vertical. The lowest area of ground around the mark is in the W-NW direction. The soil around the mark is also rocky and rocks up to 10 cm high are within 1.5 meters of the antenna. Considering how non-ideal the ground is around the mark, the amplitude pattern matches the predicted pattern rather well.

Figure 11 shows SNR values for the same satellite observed at DE01 on the previous day when the site was occupied with a high antenna setup. Trace 3 shows the filtered SNR values for L1 and trace 4 shows a synthetic amplitude for an antenna height of 1.5 meters. The actual antenna height was not exactly 1.5 meters and trace 4 is shown only to get an idea of what kind of signal would be expected for a high antenna setup. There is some significant low frequency content in the filtered SNR values which may be coming from low frequency components of the antenna gain pattern or from multipath reflections near the antenna.

Figure 11. SNR values for marker DE01 on day 158 when the point was occupied with a high antenna setup. Trace 1 shows the elevation for sv23, trace 2 shows the baseline corrected SNR values, trace 3 shows the filtered SNR values and trace 4 shows the synthetic SNR values.



The most important fact that arises out of the simple model of the multipath for the low setup is that in the elevation range of 17.5 to 29 degrees, the phase multipath error correlates strongly with a tropospheric signal. This fact is aggravated by two factors:

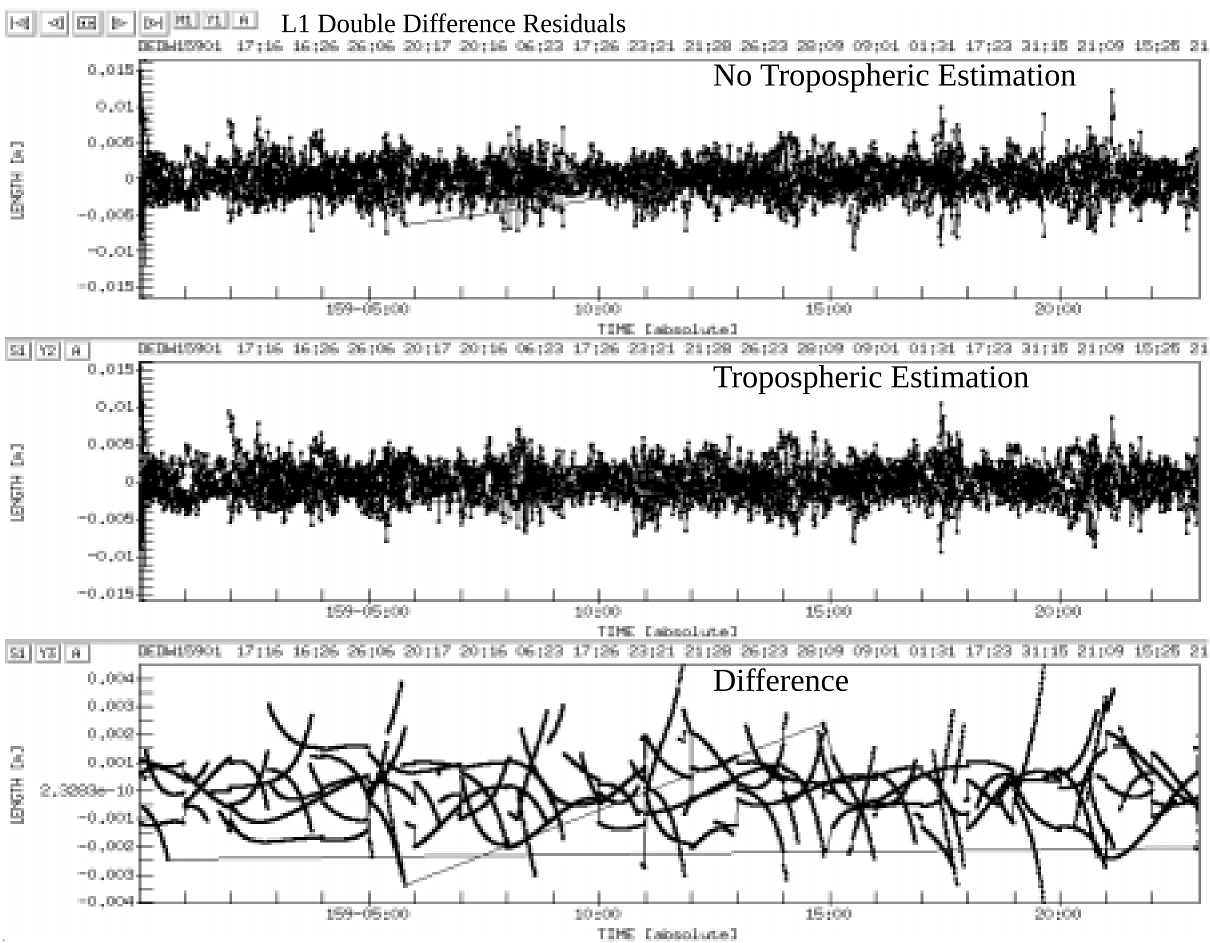
- Multipath effects are highest at lower elevations because the direct signal is entering the antenna at a point where the gain is relatively low, and the reflected signal is entering the antenna where the gain is relatively high. Therefore, the ratio of the amplitude of the reflected signal to the direct signal is high [e.q. *Schupler et al.*, 1994; *Clark*, 1992].
- Data at this elevation are weighted 3 times higher for tropospheric modeling than data at higher elevations. This is because the tropospheric signal is strongest at lower elevations.

Double Difference Residuals

Some additional insight may be gained by looking at the double difference residuals for solutions with and without tropospheric estimation. To make it easier to map synthetic multipath patterns to observations, we will look at residuals for L1, since there is a large difference for the solutions at this frequency with and without tropospheric estimation.

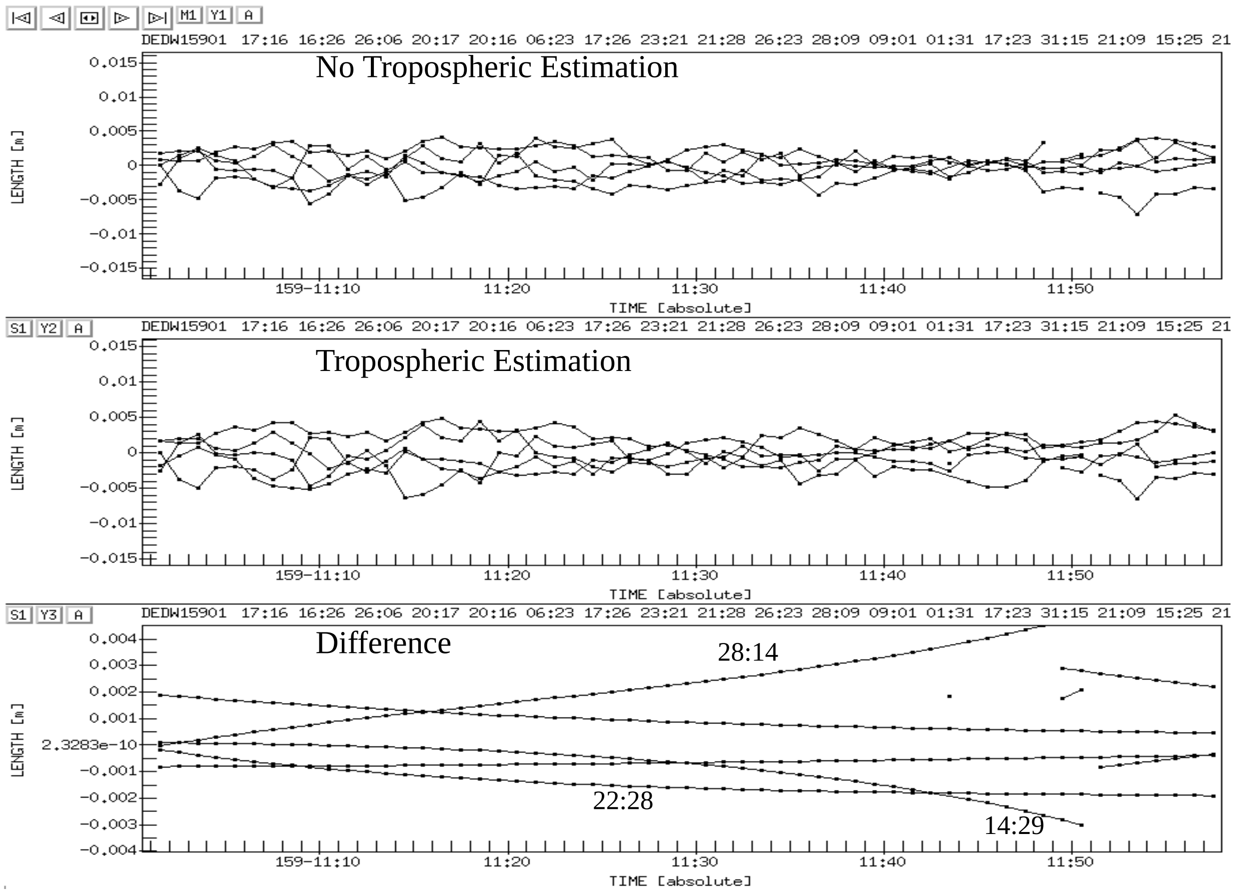
Figure 12 shows the double difference residuals for day 159 with and without tropospheric estimation. The top trace shows residuals without tropospheric estimation and the middle shows residuals with tropospheric estimation. It is difficult to see the changes between the first and second traces and they look very similar. The third trace shows the difference between the top two residuals and has a peak to peak range of 8 mm. If we look at the top two traces carefully, we can see that the residuals with tropospheric estimation are more tightly scattered around zero. For example, the excursions in the no tropospheric residuals at hour 21:00 are pulled in for the residuals with tropospheric estimation.

Figure 12. L1 Double difference residuals for solutions with and without tropospheric estimation. The top traces shows residuals for the no trop solution, the middle trace shows the residuals for the trop solution and the bottom shows the difference in the residuals.



The jumps in the differences between residuals at each hour are caused because a different tropospheric parameter was estimated for each hour.

Figure 13. A zoomed up version of the double difference residuals from 11:00 to 12:00.



If we zoom in on the time between 11:00 and 12:00 (Figure 13), we can take a closer look at some of the double difference pairs that show a large change between the two solution types. Here we see that the double difference pair 28:14 and 14:29 are different for the later half hour. The 28:14 double difference also seems to be affected for the first half hour as well, while 14:29 does not seem affected for the first half hour. The pair 22:28 is effected in a similar but opposite direction for the first half hour. From these observations, it seems that satellite 14 is being affected for the later half hour, satellite 28 for the first half hour, and 29 not at all. Figure 14 shows the elevations for the satellites 14, 28 and 29. Satellite 14 is setting and is in the critical elevation area for the second half of the hour. Satellite 28 is rising and is in the critical elevation area for the first 3/4 of the hour and satellite 29 is near its peak elevation for the entire period.

The fact that low elevation satellites are affected is not surprising, however, since if a tropospheric parameter is estimated, the difference in residuals will be greatest when satellites are at low elevations. It does point out that the tropospheric parameters are more sensitive to low elevation data, and errors that correlate to a 1/sine signal in this elevation range can be mismodeled as troposphere.

Figure 14. Elevations of satellites 14, 28 and 29. The top three traces show elevations for the entire 23 hour session and the bottom three traces show elevations for the time period from 11:00 to 12:00. The time axis on the lower three traces is relative to the first epoch, and thus starts at 0.0 instead of 11.0. The shaded bars in traces 4 and 5 show the elevation range between 17 and 29 degrees, which is the time that multipath phase errors will correlate with a tropospheric signal.

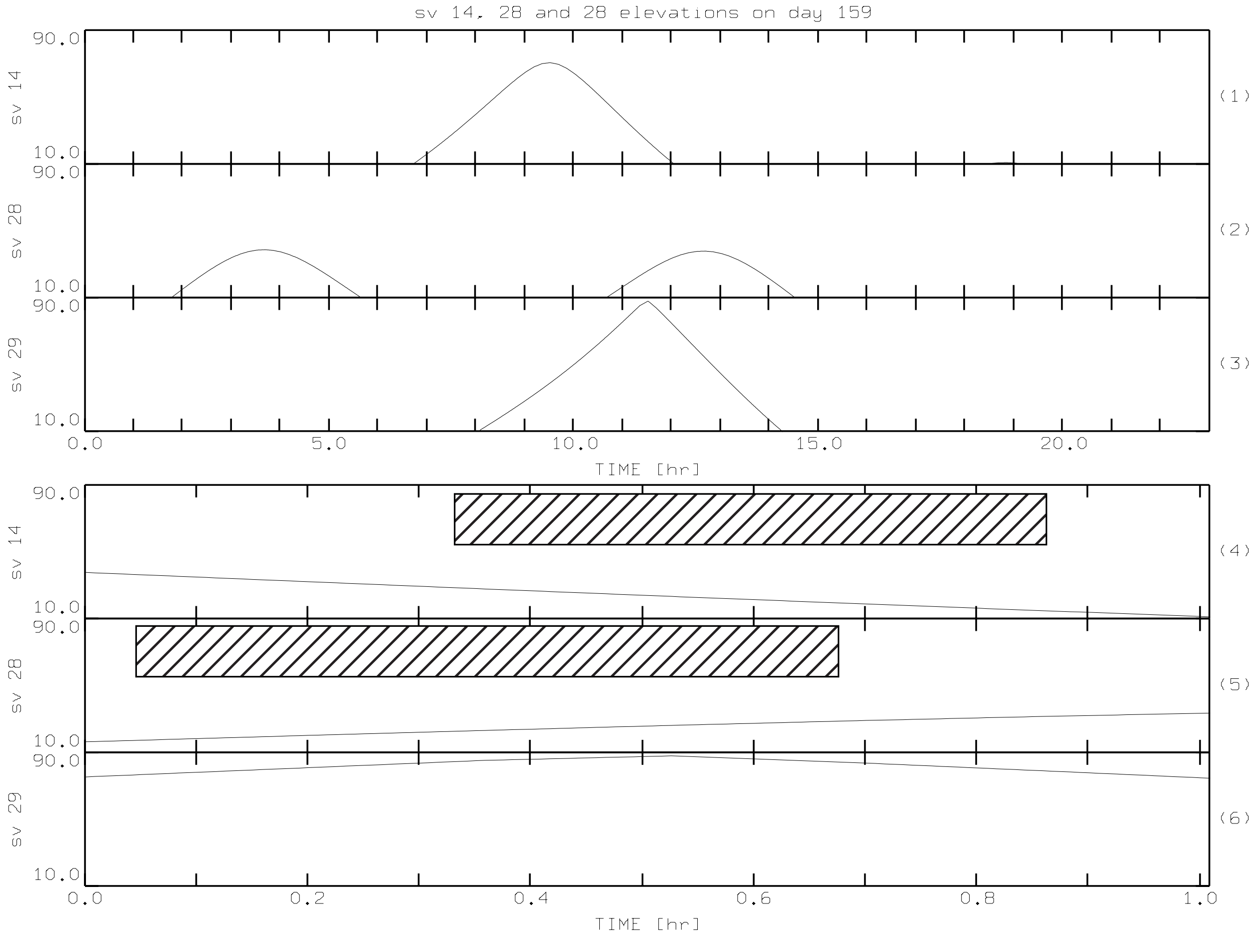
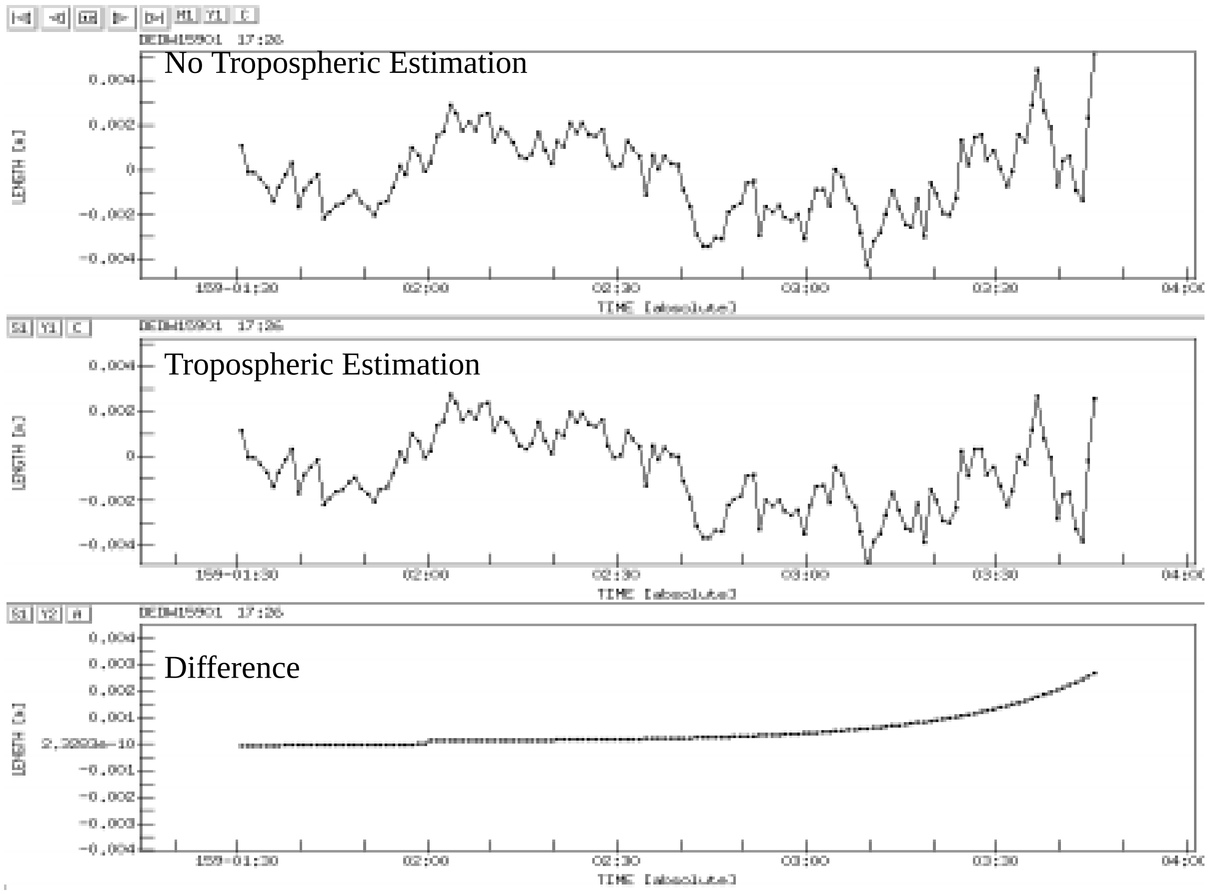


Figure 15 shows a close up view of the residuals for the satellite pair 17:26. The residuals near the end of the time series are pulled closer to zero. This shows us that with tropospheric estimation, residuals that look better are generated by the addition of a tropospheric parameter.

Figure 15. The double difference pair 17:26. The residuals in the later part of the time series are pulled closer to zero for the tropospheric estimation.



Multipath Identification

Axelrad et al. [1994], suggest that it may be possible to identify and eliminate multipath sources by analyzing SNR values of the GPS signals. They propose to identify multipath reflectors by isolating sections of the SNR data with strong spectral peaks. From the frequency of these peaks, it would then be possible to generate a model of the phase errors introduced. Their simulated and experimental results showed that it is possible to identify and even correct multipath for certain configurations.

For the low antenna setup, there is no clear frequency content, even across the entire time series. This is because the amplitude changes slowly with elevation. To make matters worse, the resulting amplitude signal is non linear and has broad spectral content. The only means to model the multipath effect would be to match exactly the entire time series with a synthetic amplitude. This is impossible for practical setups and we see little hope for determining the multipath phase errors from the SNR data from the low antenna setup at Table Mountain.

Conclusions

These preliminary results indicate:

1. SST antennas should not be mounted near to the ground (< 1.0 m), or other reflecting surfaces such as wide pillars, since the multipath phase errors can be mismodeled as a tropospheric correction. The resulting height bias between high and low SST antenna setups may be as high 2.0 cm.
2. The antenna height of 24 cm is bad because the elevation range where the multipath phase errors look like a tropospheric signal are in the range of 17.5 to 29 degrees of elevation. This elevation range is where (a) multipath is highest and (b) the tropospheric signal is strong and heavily weighted.
3. Because of the low frequency and broad frequency content (the amplitude error can not be approximated as a single sine function) of the multipath at low antenna heights, there is little hope of using SNR data to model the multipath error by looking for spectral peaks in the SNR data.
4. Antenna heights of around 1.5 meters cause multipath phase errors that cannot be modeled as a troposphere. This is because the rate of change of phase error with respect to elevation is much too high to be modeled a tropospheric delay since the phase error oscillates rapidly with elevation.
5. Further testing of other environments, receivers and antennas is required.

It is important to note that this paper only addresses multipath at the Table Mountain site, where there are no close buildings and multipath is assumed to come from ground reflections. The results and conclusions presented here may be quite different in other environments. Placing the antenna at 1.5 m and/or 24 cm will not always provide the same results, especially when multipath reflections enter the GPS antenna from elevation angles greater than zero. For these cases, the multipath effects will have different characteristics.

Suggestions for Further Work

1. Results from high-low baselines using the AOA choke ring antenna need to be analyzed.
2. Methods for suppressing multipath in the low setup should be investigated. Possibilities include (a) placing a screen around the bottom of the antenna, (b) placing microwave absorbing material underneath the antenna, and (c) increasing the number of choke rings around the antenna.
3. Testing in different environments than Table Mountain should be performed.
4. A minimum height for current GPS antenna types should be determined for various environments.
5. Development of a better geodetic antenna that not only suppresses multipath, but also has a well behaved gain and phase center pattern.

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